TORUS MODELLING

Paul McMillan

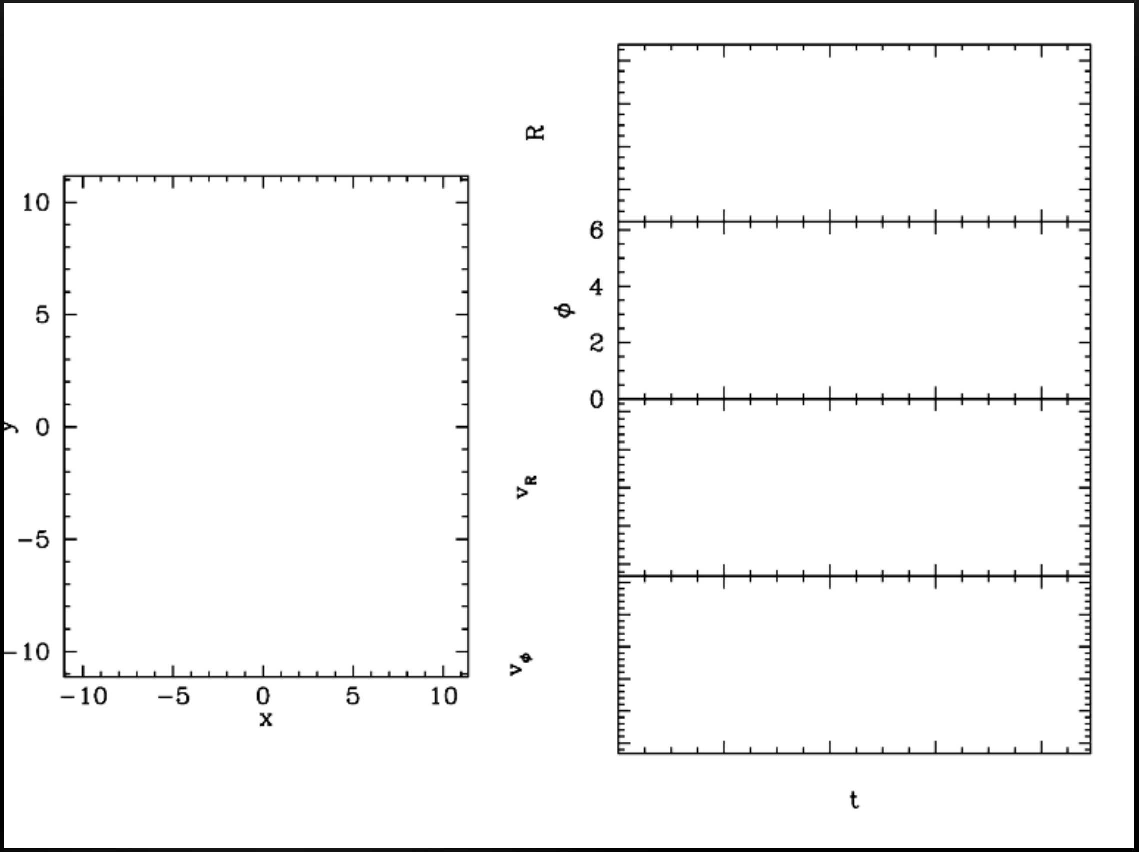
University of Oxford

ANGLE-ACTION COORDINATES

Orbits in galactic potentials are messy

Normally stored as time-series.

Better to work in coordinates in which the orbits are simple…

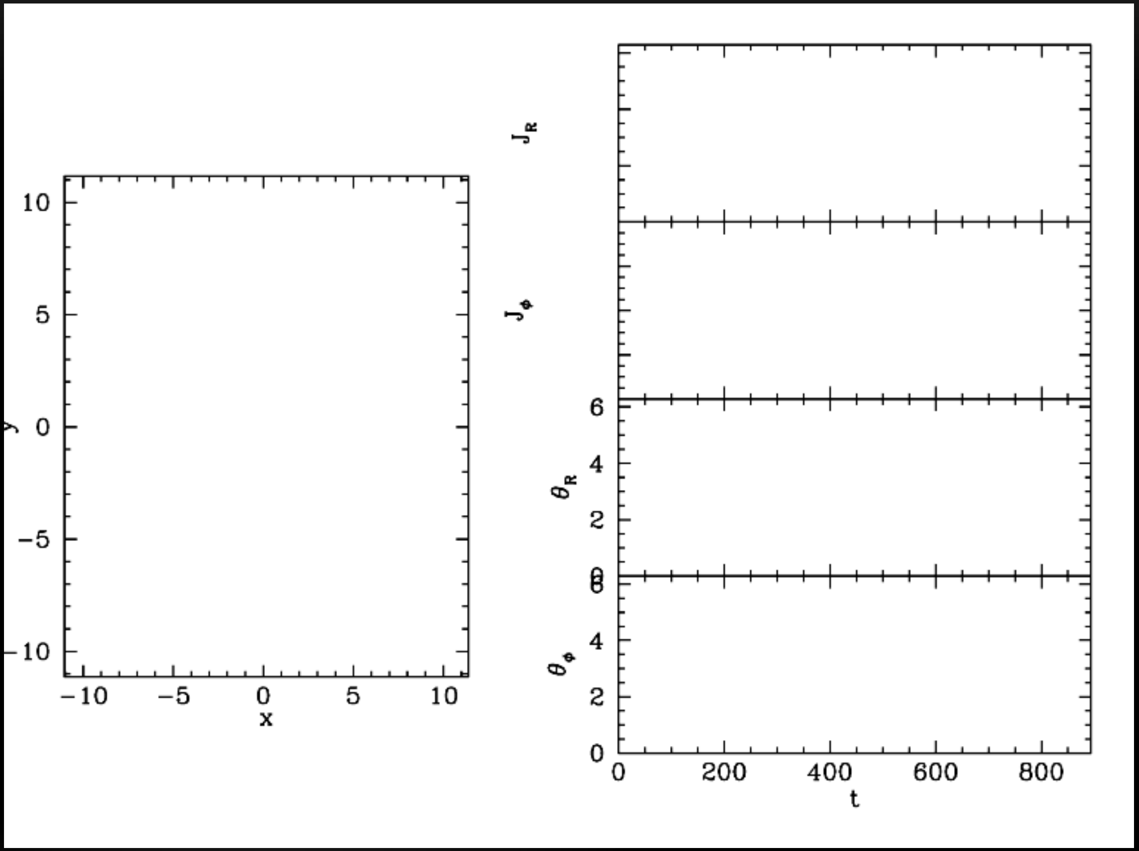


ANGLE-ACTION COORDINATES

Actions ($\underline{J}$) are constant (define/label an orbit in a meaningful way)

Angles ($\underline{\theta}$) increase linearly with time (2π-periodic)

Only known analytically for one family of potentials



BUT HOW CAN WE FIND THEM? TORUS MODELLING

McGill & Binney (1990) – We can distort the tori in a "toy" potential into our potential of interest

Ensure distorted tori are surfaces of const H and Poincare's integral vanishes on any region, U, of it.

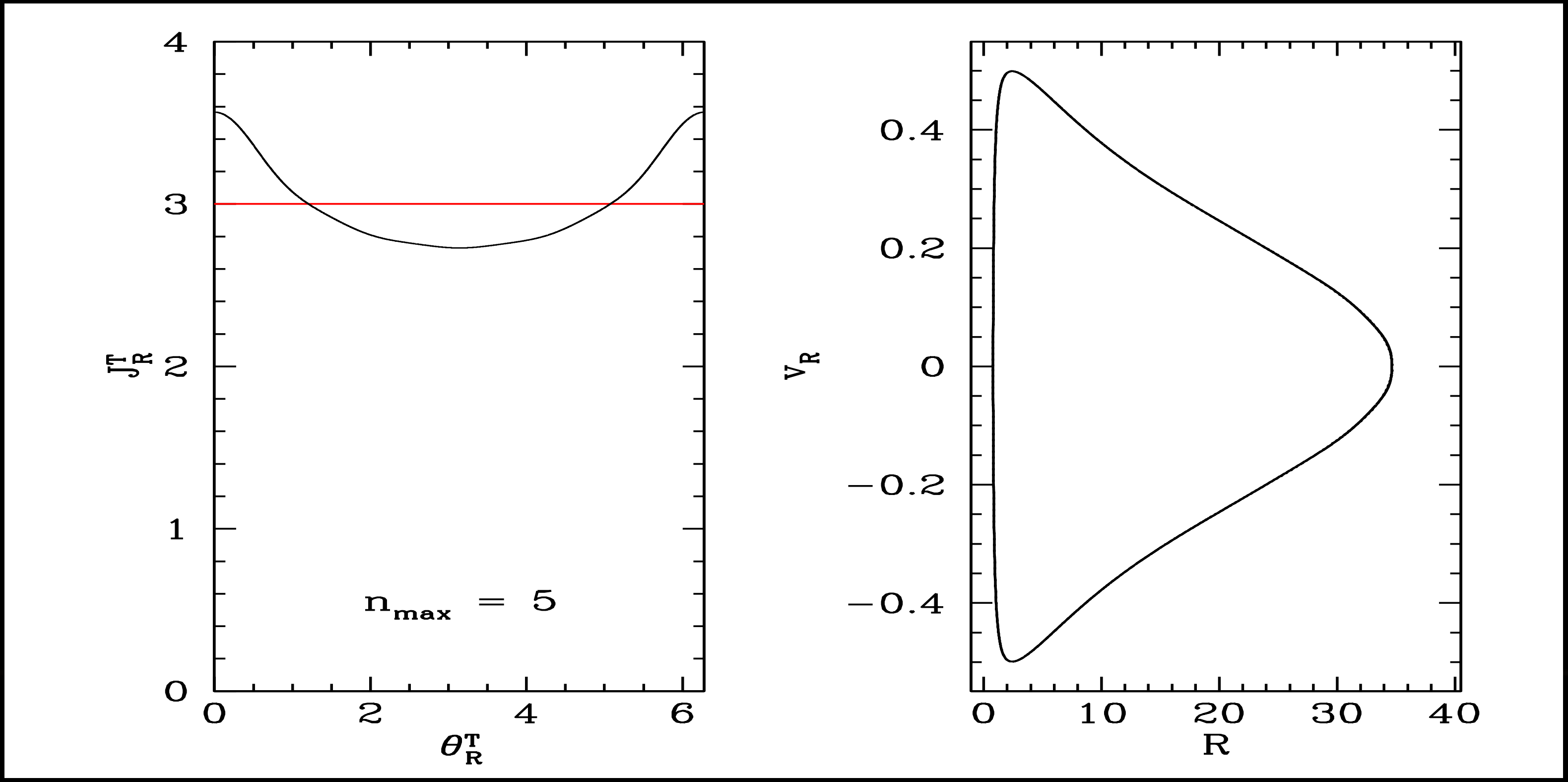
$$\int_U \mathrm{d}\mathbf{p} \cdot \mathrm{d}\mathbf{q}$$

Use a generating function – toy torus has $\underline{\boldsymbol{J}}^T$, $\underline{\boldsymbol{\theta}}^T$.

$$J_i^T = \frac{\partial\, \mathrm{S}(\mathbf{J}, \boldsymbol{\theta}^T)}{\partial \theta_i^T}; \quad \theta_i = \frac{\partial\, \mathrm{S}(\mathbf{J}, \boldsymbol{\theta}^T)}{\partial J_i}$$

BUT HOW CAN WE FIND THEM? TORUS MODELLING (CONT…)

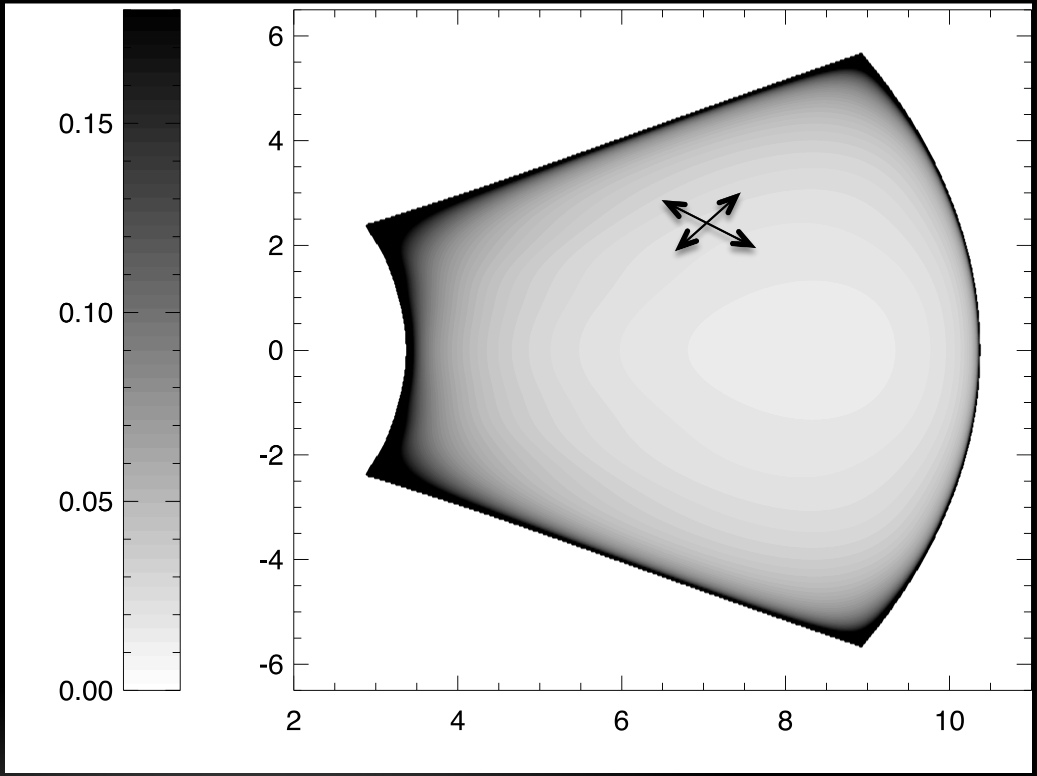
Periodicity gives constraints, so we have $\mathbf{J} = \mathbf{J}^T - \sum_{\mathbf{n}\neq 0} \mathbf{n}\, S_{\mathbf{n}}(\mathbf{J}) \cos(\mathbf{n}\cdot\boldsymbol{\theta}^T)$



TORUS VS. INTEGRATED ORBIT

Integrating an orbit: gives thousands of x,v, covers the volume sparsely.

Torus model covers entire orbit with ~ 40 parameters. Density and velocities known at any point.



BENEFITS

1. Access to $\underline{\boldsymbol{J}}$

Uniquely defines an orbit
f($\underline{\boldsymbol{J}}$) is in steady state (Jeans theorem)
$f(\underline{\mathbf{x}},\underline{\mathbf{v}})\, d^3\underline{\mathbf{x}}\, d^3\underline{\mathbf{v}} = f(\underline{\boldsymbol{J}})\, d^3\underline{\boldsymbol{\theta}}\, d^3\underline{\boldsymbol{J}}$, so orbit weight ➔ f($\underline{\boldsymbol{J}}$)
Adiabatic invariance

2. Access to $\underline{\boldsymbol{\theta}}$

Describe entire orbit (using ~40 values!)
Allows interpolation between $\underline{\boldsymbol{J}}$, perturbation theory

BUT require specialised software (currently only axisymmetric case)
Takes ~0.1s/torus for typical disc orbits

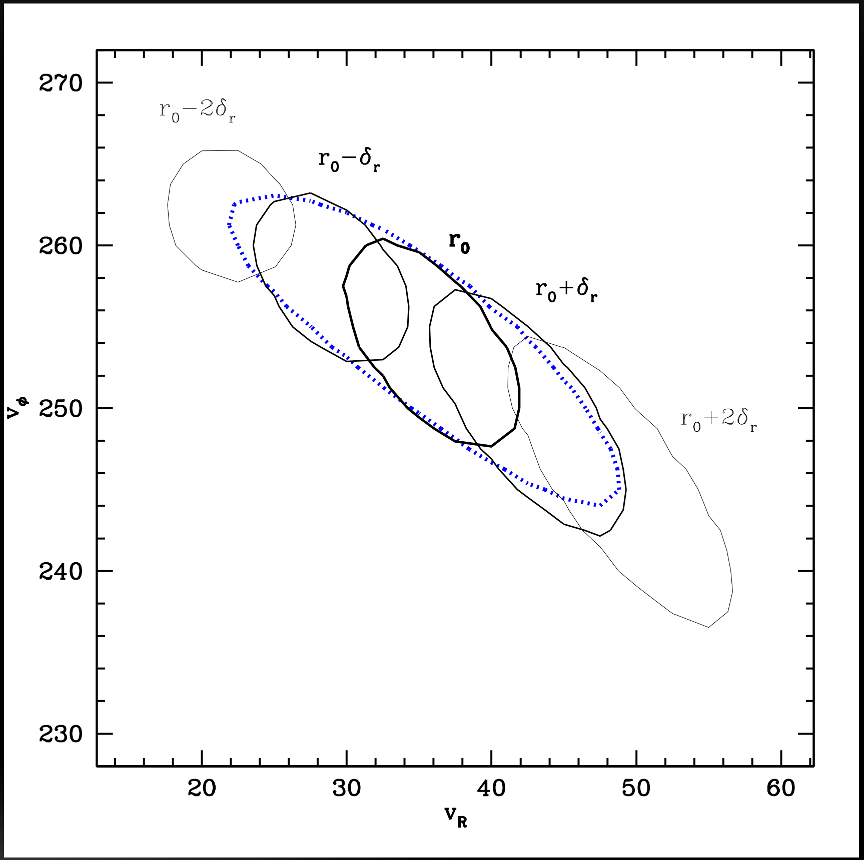
MILKY WAY OBSERVABLES

Observable/kinematic data on the Milky Way is uniquely rich

Multi-dimensional (l, b, ϖ/r_{phot}, $\boldsymbol{\mu}$, v_{los}, m, colour, g, Z, α/Fe…)

Uncertainties highly correlated in derived coordinates (e.g. v_R, v_ϕ)

Want to work in observables space, but too many dimensions to bin!



EVALUATING A MODEL *WITHOUT* BINNING

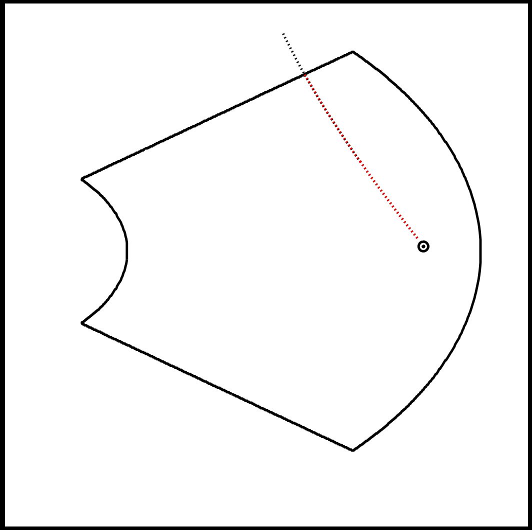
We need P(observables) directly for a given model – with tori we have this

Can sample f(***J***), as relationship f(***J***) ➔ orbit weights is known

For each ***J***, torus model provides provides P(observables|***J***) for a given star as integral along line of sight (can 'paint' each torus)

Sum probabilities (appropriately weighted by any f(***J***), and selection effects), then take product to give likelihood

Once LOS integrals done, trivial to compare dfs (harder to compare potentials) – LOS integrals take time (tori × stars)

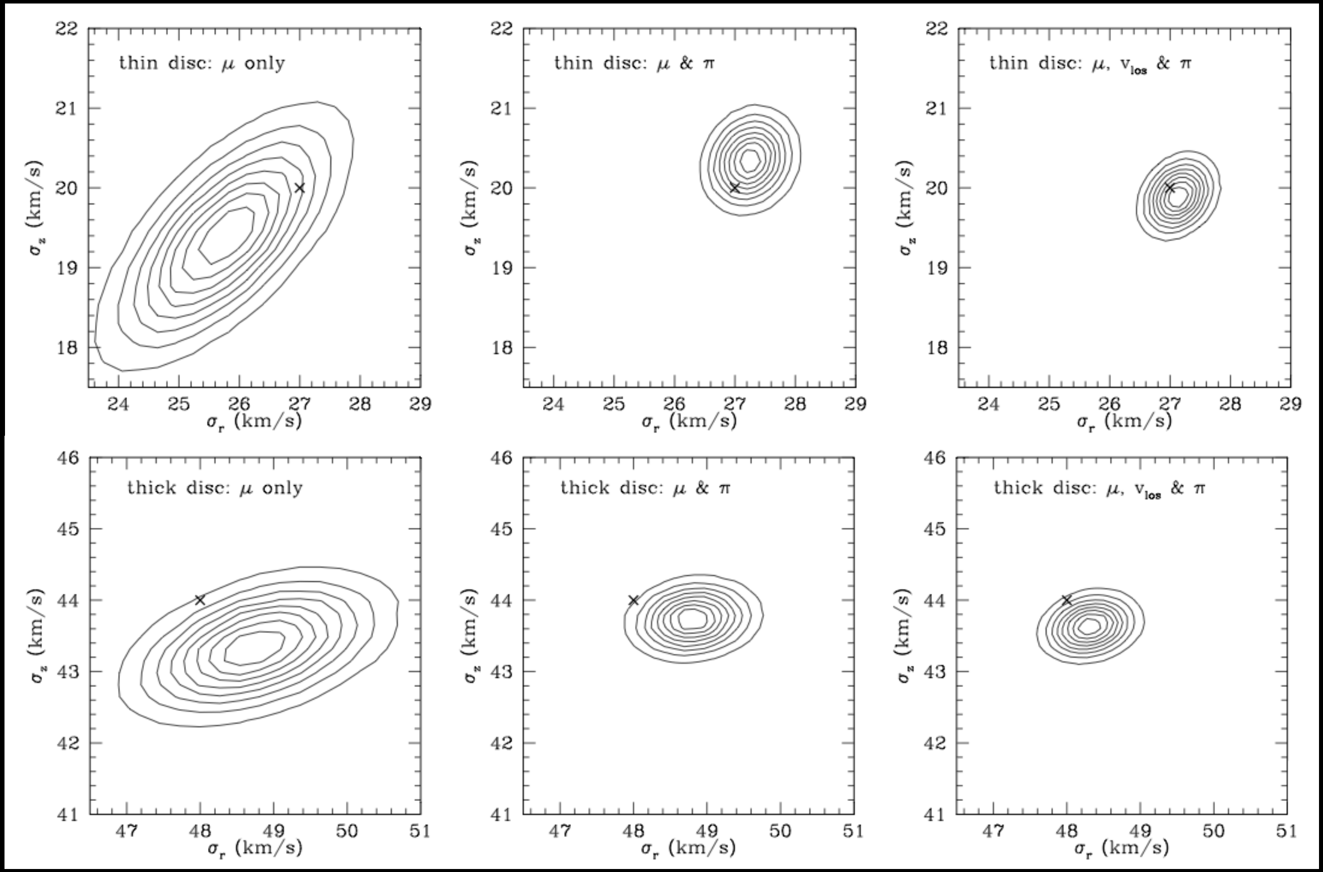


EVALUATING A MODEL *WITHOUT* BINNING

How well can we recover known f(J) from minimal observational data?

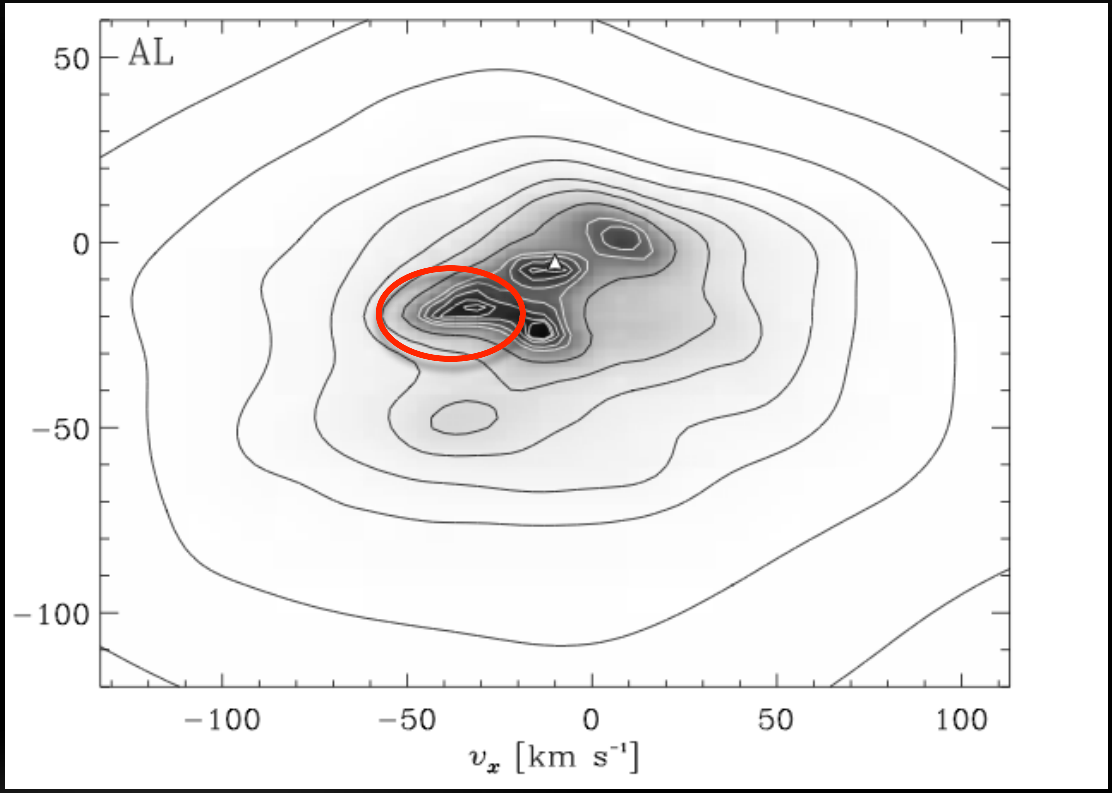
With 10000 stars, simple (magnitude) limits and (Gaia-ish) data sets of varying richness

(McMillan & Binney, 2012)



THE HYADES - THE SOLAR NEIGHBOURHOOD

Moving group in the solar neighbourhood (e.g. Dehnen 98)

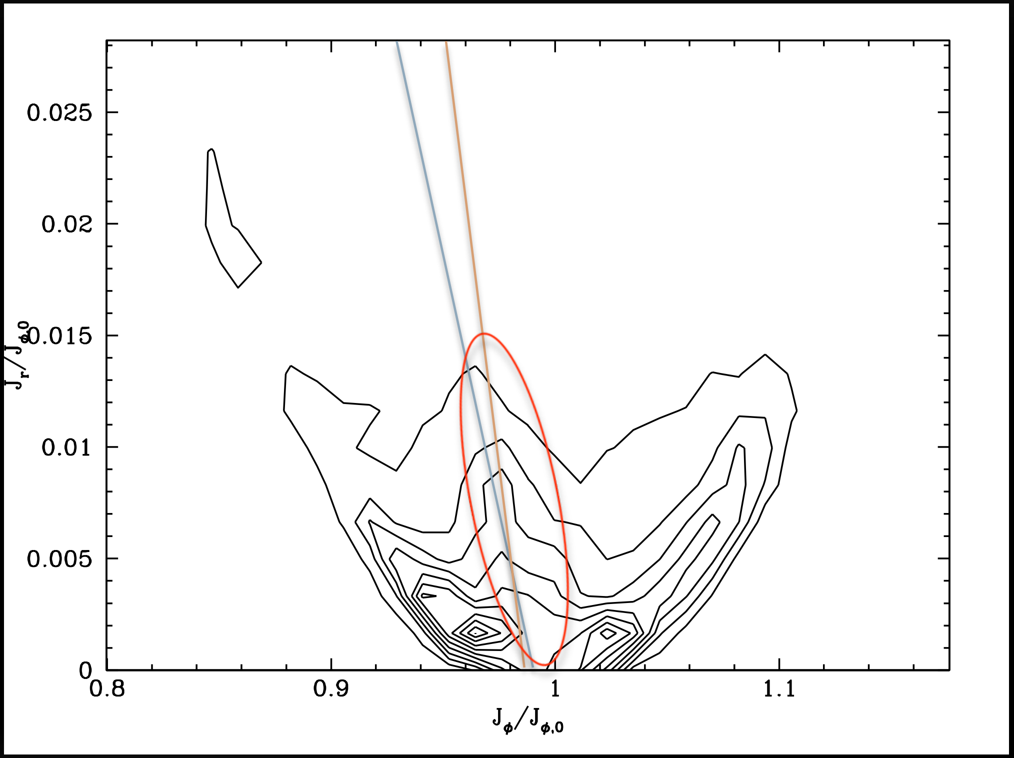


THE HYADES - THE SOLAR NEIGHBOURHOOD

Moving group in the solar neighbourhood (e.g. Dehnen 98)

Sellwood (2010) noted that it showed up in action space along resonance lines

$$l\Omega_r + m\Omega_\phi = \mathrm{const} = m\Omega_\mathrm{p}$$



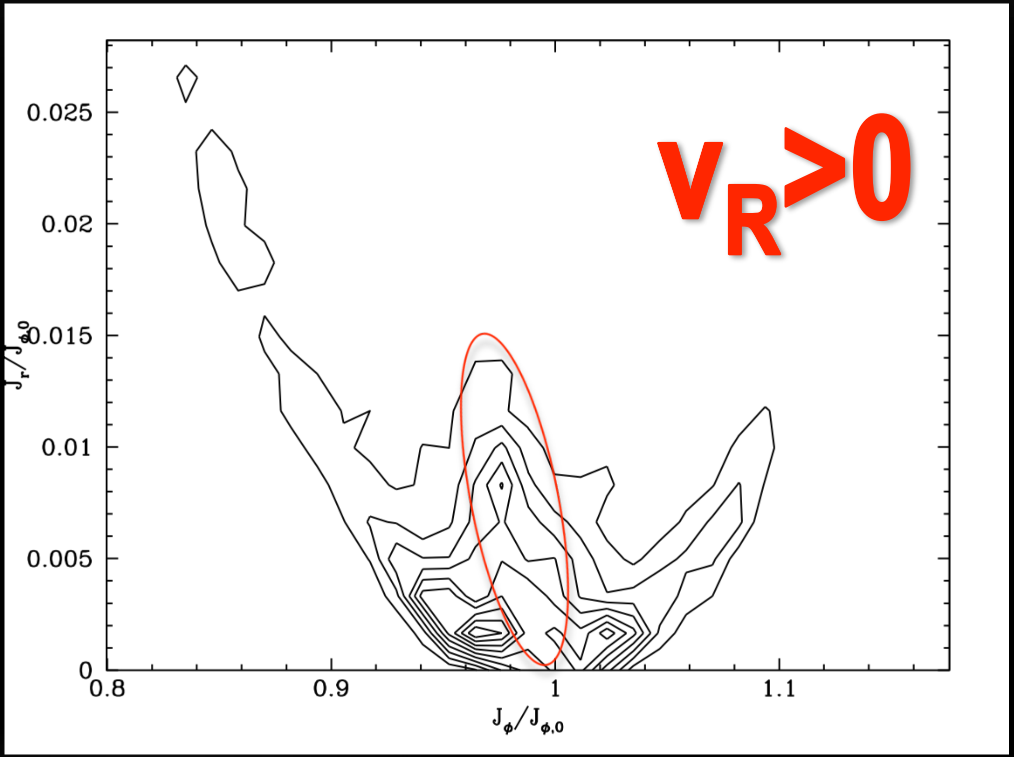
THE HYADES - THE SOLAR NEIGHBOURHOOD

Moving group in the solar neighbourhood (e.g. Dehnen 98)

Sellwood (2010) noted that it showed up in action space along resonance lines

$$l\Omega_r + m\Omega_\phi = \text{const} = m\Omega_\text{p}$$

But only for $v_R > 0$



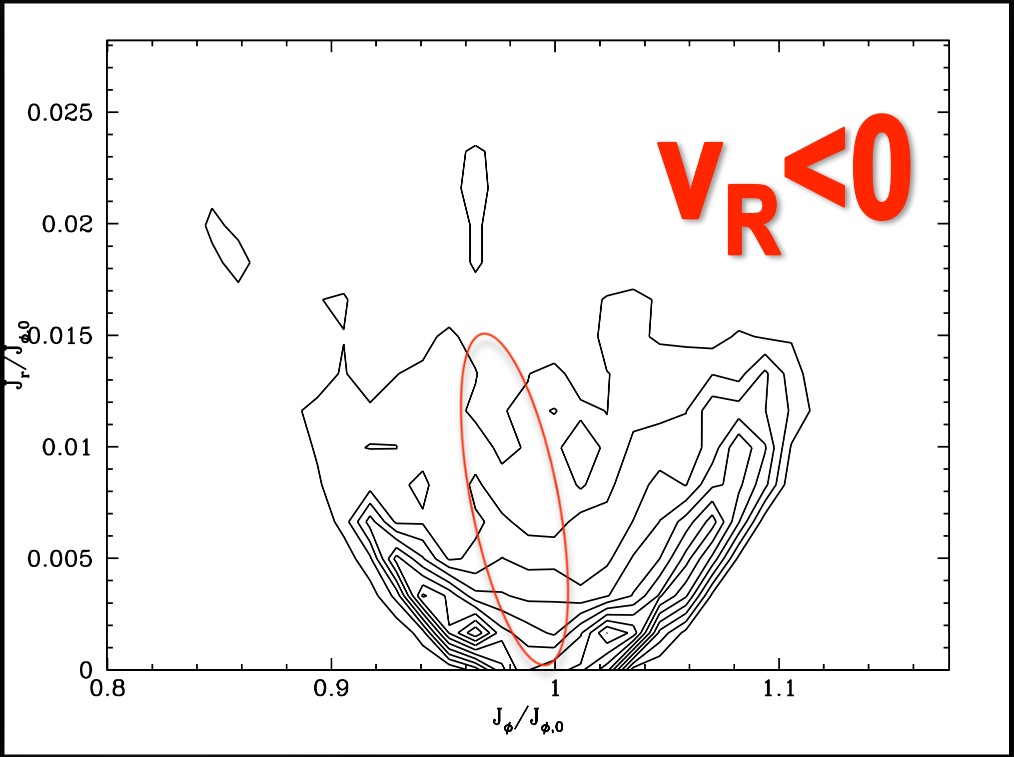
THE HYADES - THE SOLAR NEIGHBOURHOOD

Moving group in the solar neighbourhood (e.g. Dehnen 98)

Sellwood (2010) noted that it showed up in action space along resonance lines

$$l\Omega_r + m\Omega_\phi = \mathrm{const} = m\Omega_\mathrm{p}$$

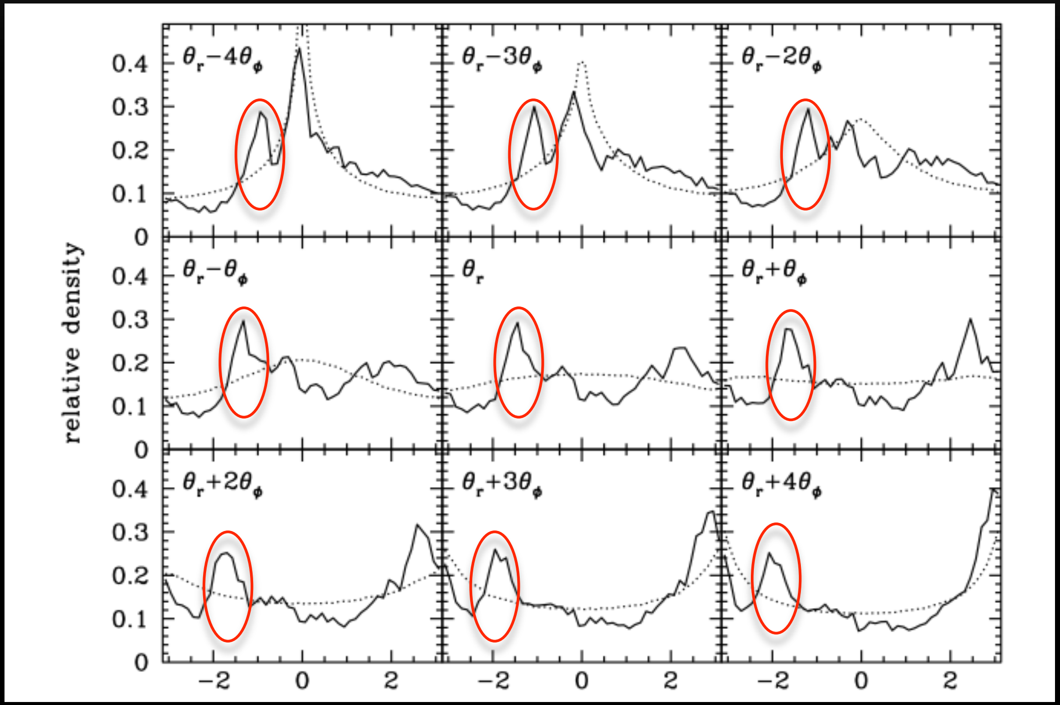
But only for $v_R > 0$



THE HYADES - THE SOLAR NEIGHBOURHOOD

Expect stars that were trapped to have

$$l\theta_r + m\theta_\phi \simeq \text{const},$$



Problem: selection effects (small volume) – for given J, limits possible θ.

But:

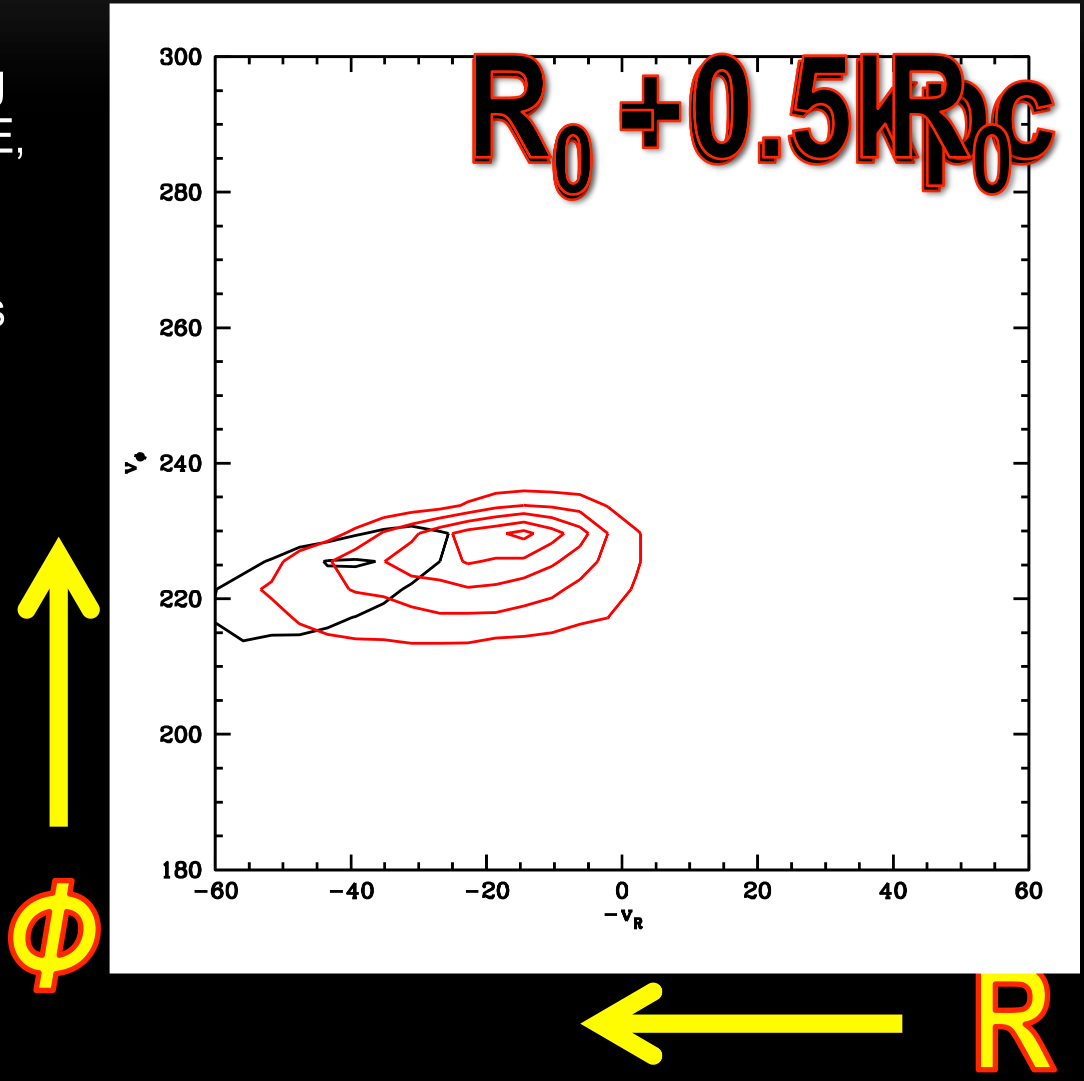
Simple model, no assumptions about perturbation, easily tunable.

THE HYADES – BEYOND THE SN

Data beyond the SN is becoming available (Gaia, Segue, APOGEE, RAVE)

Torus model provides predictions for what we should see in these fields

Potentially allow us to discriminate between different resonances – easy to test many.



CONCLUSIONS

Torus models are a method of accessing angle-action coordinates for general potentials

Makes dynamics easy

Can provide full probability density associated with an orbit (and thus a dynamical model)

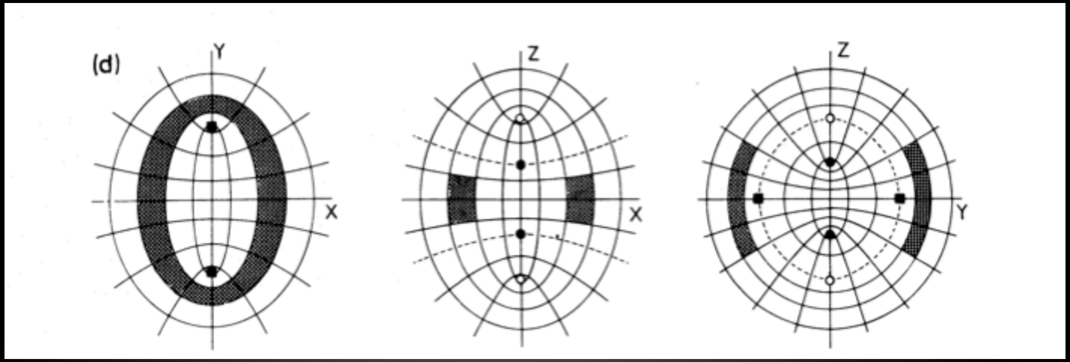
Can provide simple tunable models of the Hyades to compare to data.

BUT HOW CAN WE FIND THEM? ANALYTICALLY?

Only for isochrone potential (spherically symmetric)

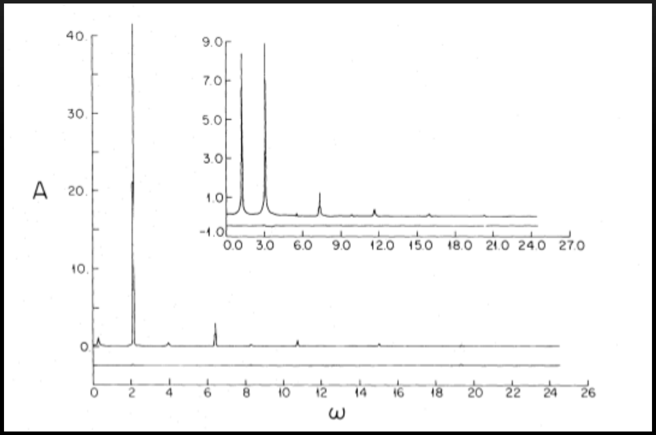
$$\Phi(r) = -\frac{GM}{b + \sqrt{b^2 + r^2}}$$

(Stäckel potential – integrals of motion analytic, angle-actions require numerical integration, de Zeeuw 1985)

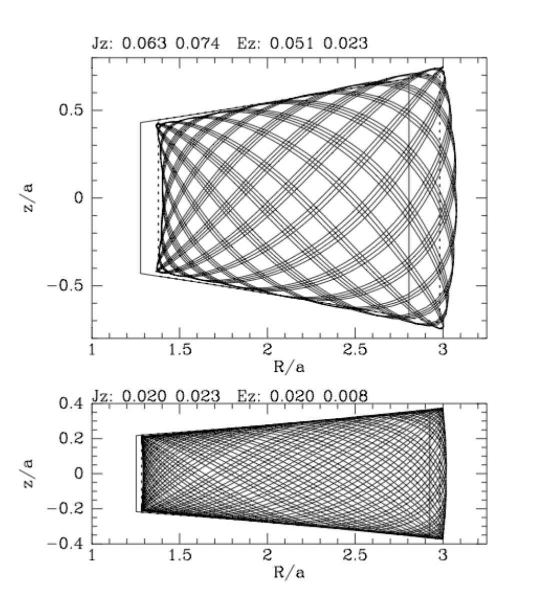


BUT HOW CAN WE FIND THEM? NUMERICALLY?

Spectral dynamics (Binney & Spergel 1982,4) - Good for frequencies – actions trickier



Adiabatic approximation (e.g. Binney 2010, Binney & McMillan 2011, Binney in prep) – Good for disc models (with some limitations).



Other possibilities…

HISTORY

McGill & Binney (1990, MNRAS 244, 634)

Binney & Kumar (1993, MNRAS 261, 584)

Kaasalainen & Binney (1994a, MNRAS, 268, 1033)

Kaasalainen (1994, MNRAS, 268, 1041)

Kaasalainen & Binney (1994b, PhRvL, 73, 2377)

Kaasalainen (1995a, MNRAS, 275,162)

Kaasalainen (1995b, PhRvE, 52, 1193)

Dehnen & Binney (1996, ASPC, 92, 393)

…

McMillan & Binney (2008, MNRAS,390,429)

Binney & McMillan (2011, MNRAS, 413, 1889)

McMillan (2011, MNRAS, 418, 1565)

McMillan & Binney (2012, MNRAS, 419, 2251)